

**NDU**

**MAT 224**

**Calculus IV**

**Exam # 2**

**Thursday May 29, 2014**

**Duration: 60 minutes**

**Name:** \_\_\_\_\_

**Section:** \_\_\_\_\_

**Grade:** \_\_\_\_\_

Problem Number	Points	Score
1	40	
2	13	
3	18	
4	32	
Total	103	

- 1) (40 points) For each of the following multiple-choice questions, circle the **letter** of the correct answer. If more than one letter is circled in the same problem, you will receive no credit for that problem.

**Question A (8 points)** Using the method of Lagrange Multipliers to find the point closest to the origin on the curve of intersection of the plane  $x + y + z = 1$  and the cone  $z^2 = 2x^2 + 2y^2$ , can yield the system of equations:

- |                          |                         |                          |                          |
|--------------------------|-------------------------|--------------------------|--------------------------|
| a) $2x = \lambda + 4\mu$ | b) $2x = 4\mu$          | c) $2x = \lambda + 4\mu$ | d) $2x = \lambda + 4\mu$ |
| $2y = \lambda + 4\mu$    | $2y = 4\mu$             | $2y = \lambda + 4\mu$    | $2y = \lambda + 4\mu$    |
| $2z = \lambda$           | $2z = \lambda$          | $2z = \lambda - 2\mu$    | $2z = \lambda - 2\mu$    |
| $x + y + z - 1 = 0$      | $x + y + z - 1 = 0$     | $x + y + z = 0$          | $x + y + z - 1 = 0$      |
| $2x^2 + 2y^2 - z^2 = 0$  | $2x^2 + 2y^2 - z^2 = 0$ | $2x^2 + 2y^2 = 0$        | $2x^2 + 2y^2 - z^2 = 0$  |

e) If none of the above is correct, circle (e) and write your answer here:

**Question B (8 points)**

$$\int_0^9 \int_{\sqrt{y}}^3 3 \sec^2(x^3) dx dy =$$

- a)  $\tan(27)$   
 b)  $\tan(8)$   
 c)  $\tan(1)$   
 d) 0  
 e) If none of the above is correct, circle (e) and write your answer here:

**Question C (8 points)**

Let  $R$  be the region in the first quadrant of the  $xy$ -plane that lies outside the circle  $x^2 + y^2 = 1$  and inside the circle  $x^2 + y^2 = 9$ . Then  $\iint_R (x + y) dx dy =$

- a)  $\frac{2}{3}$   
 b)  $\frac{52}{3}$   
 c)  $\frac{54}{3}$   
 d) 0  
 e) If none of the above is correct, circle (e) and write your answer here:

**Question D (16 points)** Consider the region D in space that is bounded from below by the  $xy$ -plane, from above by the paraboloid  $z = x^2 + y^2$  and laterally by the cylinder  $x^2 + y^2 = 4$ . Then a triple integral, in cylindrical coordinates, representing the volume of D is:

**Part 1** According to the order of integration  $dzdrd\theta$

a)  $\int_0^{2\pi} \int_0^2 \int_{r^2}^4 r dz dr d\theta$

b)  $\int_0^{2\pi} \int_0^2 \int_0^{x^2+y^2} r dz dr d\theta$

c)  $\int_0^{2\pi} \int_0^2 \int_0^{r^2} r dz dr d\theta$

d)  $\int_0^{2\pi} \int_0^4 \int_0^{r^2} r dz dr d\theta$

e) If none of the above is correct, circle (e) and write your answer here:

**Part 2** According to the order of integration  $drdzd\theta$

a)  $\int_0^{2\pi} \int_0^4 \int_0^{\sqrt{z}} r dr dz d\theta$

b)  $\int_0^{2\pi} \int_0^4 \int_{r^2}^2 r dr dz d\theta$

c)  $\int_0^{2\pi} \int_0^2 \int_{\sqrt{z}}^4 r dr dz d\theta$

d)  $\int_0^{2\pi} \int_0^{r^2} \int_0^2 r dr dz d\theta$

e) If none of the above is correct, circle (e) and write your answer here:

2) **(13 points)** Let  $R$  be the region in the first quadrant bounded by the curves  $y = 0$  and  $(x - 2)^2 + y^2 = 4$ , with  $0 \leq x \leq 2$ .

a) **(3 points)** Draw the region  $R$  in the  $xy$ -plane.

b) **(10 points)** Set up a double integral in polar coordinates using the order of integration  $drd\theta$  equal to  $\iint_R f(x, y) dA$ , where  $f(x, y) = x^4 + y^3$ .



**3) (18 points)** Let  $D$  represent the region in the first octant bounded by the coordinate planes and the planes  $3x + z = 6$  and  $y + z = 6$ . Set up triple integrals in rectangular coordinates representing the volume of  $D$  according to each of the following orders:

**a) (8 points)**  $dy dx dz$

**b) (10 points)**  $dz dy dx$

- 4) (32 points) Let  $D$  be the solid region in space bounded **from below** by the plane  $z = 3$  and **from above** by the sphere  $x^2 + y^2 + (z - 2)^2 = 4$ .
- a) (4 points) Draw the region  $D$ .

b) (5 points) Rewrite each of the above equations using spherical coordinates.

c) (10 points) Set up triple integrals in spherical coordinates for the volume of  $D$  using the order of integration  $d\rho d\phi d\theta$ .

**d) (13 points)** ) Set up triple integrals in spherical coordinates for the volume of  $D$  using the order of integration  $d\varphi d\rho d\theta$ .